

## 6.2 Closed-Loop Oscillation-Based Tuning

A PID controller has three tuning parameters. If these are adjusted in an ad hoc fashion, it may take a while for satisfactory performance to be obtained. Also, each tuning technician will end up with a different set of tuning parameters. There is plenty of motivation, then, to develop an algorithmic approach to controller tuning. The first widely used method for PID tuning was published by Ziegler and Nichols in 1942.

### Ziegler-Nichols Closed-Loop Method

The Ziegler-Nichols closed-loop tuning technique was perhaps the first rigorous method to tune PID controllers. The technique is not widely used today because the closed-loop behavior tends to be oscillatory and sensitive to uncertainty. We study the technique for historical reasons, and because it is similar to commonly used automatic tuning ("auto-tune") techniques covered in Chapter 11.

The closed-loop Ziegler-Nichols method consists of the following steps.

1. With P-only closed-loop control, increase the magnitude of the proportional gain until the closed-loop is in a continuous oscillation. For slightly larger values of controller gain, the closed-loop system is unstable, while for slightly lower values the system is stable.
2. The value of controller proportional gain that causes the continuous oscillation is called the critical (or ultimate) gain,  $k_{cu}$ . The peak-to-peak period (time between successive peaks in the continuously oscillating process output) is called the critical (or ultimate) period,  $P_u$ .
3. Depending on the controller chosen, P, PI, or PID, use the values in Table 6-1 for the tuning parameters, based on the critical gain and period.

Tyreus and Luyben have suggested tuning parameter rules that result in less oscillatory responses and that are less sensitive to changes in the process condition. Their rules are shown in Table 6-2.

**Table 6-1** Ziegler-Nichols Closed-Loop Oscillation Method Tuning Parameters

Controller type	$k_c$	$\tau_I$	$\tau_D$
P-only	$0.5 k_{cu}$	—	—
PI	$0.45 k_{cu}$	$P_u/1.2$	—
PID	$0.6 k_{cu}$	$P_u/2$	$P_u/8$

## 6.2 Closed-Loop Oscillation-Based Tuning

**Table 6-2** Tyreus-Luyben Suggested Tuning Parameters Based on the Ziegler-Nichols Closed-Loop Oscillation Tuning Method

Controller type	$k_c$	$\tau_I$	$\tau_D$
PI	$k_{cu}/3.2$	$2.2 P_u$	—
PID	$k_{cu}/2.2$	$2.2 P_u$	$P_u/6.3$

Since first-order + time-delay processes have a maximum slope of  $k = k_p/\tau_p$  at  $t = \theta$  for a unit step input change, these same rules can be used for first-order + time-delay processes,

$$g_p(s) = \frac{k_p e^{-\theta s}}{\tau_p s + 1}$$

Their recommended tuning parameters, which should give roughly quarter-wave damping, are shown in Table 6-3. We see a potential problem for systems with a low time-delay/time-constant ratio, since this causes the proportional gain to become very large. Similarly, the integral time tends to be low, causing oscillatory behavior.

### Cohen-Coon Parameters

The method developed by Cohen and Coon (1953) is based on a first-order + time-delay process model. A set of tuning parameters was empirically developed to yield a closed-loop response with a decay ratio of 1/4 (similar to the Ziegler-Nichols methods). The tuning parameters as a function of the model parameters are shown in Table 6-4.

A major problem with the Cohen-Coon parameters is that they tend not to be very robust; that is, a small change in the process parameters can cause the closed-loop system to become unstable.

**Table 6-3** Ziegler-Nichols Open-Loop Tuning Parameters

Controller type	$k_c$	$\tau_i$	$\tau_D$
P-only	$\frac{1}{k\theta}$ or $\frac{\tau_p}{k_p\theta}$	—	—
PI	$\frac{0.9}{k\theta}$ or $\frac{0.9\tau_p}{k_p\theta}$	3.3 $\theta$	—
PID	$\frac{1.2}{k\theta}$ or $\frac{1.2\tau_p}{k_p\theta}$	2 $\theta$	0.5 $\theta$

### 6.4 Direct Synthesis

203

**Table 6-4** Cohen-Coon Tuning Parameters

Controller type	$k_c$	$\tau_i$	$\tau_D$
P-only	$\frac{\tau_p}{k_p\theta} \left[ 1 + \frac{\theta}{3\tau_p} \right]$	—	—
PI	$\frac{\tau_p}{k_p\theta} \left[ 0.9 + \frac{\theta}{12\tau_p} \right]$	$\frac{\theta \left[ 30 + 3 \frac{\theta}{\tau_p} \right]}{9 + 20 \frac{\theta}{\tau_p}}$	—
PID	$\frac{\tau_p}{k_p\theta} \left[ \frac{4}{3} + \frac{\theta}{4\tau_p} \right]$	$\frac{\theta \left[ 32 + 6 \frac{\theta}{\tau_p} \right]}{13 + 8 \frac{\theta}{\tau_p}}$	$\frac{4\theta}{11 + 2 \frac{\theta}{\tau_p}}$

Tuning

OR  
time-delaye damp-  
w time-  
y e.

ne-delay